Teleportation of an arbitrary multipartite state via photonic Faraday rotation

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Abstract. We propose a practical scheme for deterministically teleporting an arbitrary multipartite state, either product or entangled, using Faraday rotation of the photonic polarization. Our scheme, based on the input-output process of single-photon pulses regarding cavities, works in low-Q cavities and only involves virtual excitation of the atoms, which is insensitive to both cavity decay and atomic spontaneous emission. Besides, the Bell-state measurement is accomplished by the Faraday rotation plus product-state measurements, which could much relax the experimental difficulty to realize the Bell-state measurement by the CNOT operation.

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1. Introduction

Teleportation is the faithful transfer of quantum states between spatially separated parties, based on the prior establishment of entanglement and a classical communication [1]. As a practical application, teleportation has been implemented experimentally in several quantum systems [2, 3], and could be very useful for achieving quantum repeaters [4], quantum networks [5], and also quantum computing [6]. Recently, much attention has been paid on teleportation of multipartite states [7, 8, 9]. It has been shown that an arbitrary two-qubit state can be teleported using the multipartite entangled state as quantum channel [9]. On the other hand, cavity QED system gives a very nice platform to accomplish quantum teleportation. Many schemes have been proposed in this system. The earlier schemes use the atom as a flying qubit to transfer quantum information [10, 11], which are actually unsuitable for long-distance communication. Furthermore, the schemes mentioned above work well only in high-Q cavities, which are hard to accomplish with current technology. Besides, most of those schemes, e.g., [8, 11], are intrinsically probabilistic.

Can we achieve teleportation with low-Q cavities? More recently, schemes using photons to transfer information and atoms to store information were proposed [12]. These schemes are based on the detection of photons leaking out of cavities. Taking the cavity damping into account, these schemes work in the low-Q cavity regime. However, the cavity damping actually plays a detrimental role in the success probability of the teleportaion. Thus these schemes are intrinsically probabilistic. It shows the strong dependence of the implementation on the dissipative rate. This implies a negligible possibility to achieve a teleportation with the low-Q cavities.

In this work, however, we will show this possibility with a practical scheme to teleport multipartite states using sophisticated low-Q cavities. Inspired by recent experiments of photonic input-output process [3, 13], the key idea is to make use of the Faraday rotation produced by single-photon-pulse input and output process regarding low-Q cavities. We had noticed a previous idea for teleportation using Faraday rotation by single photons from microcavities at one location to those at another location in quantum dot system [14]. While the photons input and output from the microcavities correspond to some decay effects, there was little discussion about the impact from dissipation on the teleportation. In our point of view, an efficient implementation of the teleportation in [14] requires high rate of the photons input and output with respect to the cavities. This implies the necessity of employing bad cavities with large decay rates, which goes beyond the model there.

The key point of our present scheme is to make use of a new reflection coefficient equation published recently [15], which enables us to get a rotation of the photonic polarization, named Faraday rotation [16], conditional on the atomic state confined in the cavity even in low-Q regime. We argue that our scheme would be advantageous over previous schemes [12, 14] in accomplishment of the teleportation in a deterministic fashion with currently available cavity QED technology, such as the microtoroidal

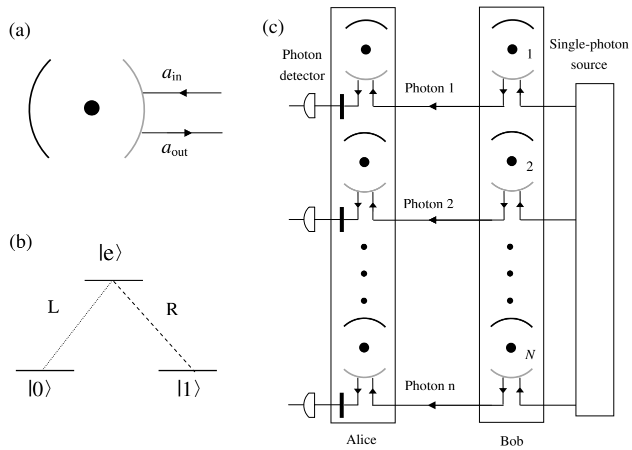

Figure 1. (a) Single photon input and output regarding the low-Q cavity. (b) The level structure of the atom confined in the cavity. The transitions from the degenerate doublet ground states $|0\rangle$ and $|1\rangle$ to the excited state $|e\rangle$ are triggered by a L - and R - circularly polarized photons, respectively. (c) Schematic for teleporting an unknown multipartite state of atoms, where the arrows show the flying direction of the photons, and the bold lines are QWPs defined in the text.

resonator [13] or the single-sided optical cavity [17]. Furthermore, due to the disentangling action of Faraday rotation to the photonic and atomic state in the destination side, only product-state measurement is needed in our scheme, which much relaxes the experimental difficulty.

This paper is organized as follows. In Sec. 2, we first briefly review the input-output process of a single-photon pulse regarding a low-Q cavity and show the Faraday Rotation of the photonic polarization [15]. In Sec. 3 we explicitly elucidate our teleportation scheme of multipartite state via Faraday rotation. The discussion about the experimental feasibility of our scheme and a short conclusion are made in Sec. 4.

2. Faraday rotation in cavity QED system

We first briefly review the input-output relation in a low-Q cavity discussed in [15]. Consider a bimodal cavity with one of the mirrors partially reflective so that the photon can be injected in and then reflected out with a certain probability, as shown in Fig. 1 (a). The level structure of the atom confined in the cavity is depicted in Fig. 1 (b), where $|0\rangle$ and $|1\rangle$ correspond to the degenerate Zeeman sublevels of the ground state in an alkali atom, and $|e\rangle$ is the excited state. The atomic transitions $|e\rangle \leftrightarrow |0\rangle$ and $|e\rangle \leftrightarrow |1\rangle$ are due to the coupling of the atom to the two degenerate cavity modes a_L (with left circular polarization) and a_R (with right circular polarization), respectively. Each of the transition channels is governed by the Hamiltonian of usual Jaynes-Cummings model.

Suppose a L - (or R -) circularly polarized single-photon pulse with the frequency ω_p is input into the cavity. The pulse is denoted by $|\psi\rangle = \int_0^T f(t)a_{in}^{\dagger}(t)dt|vac\rangle$, where $f(t)$ is a time-dependent normalized pulse shape with the pulse duration T, $a_{in}^{\dagger}$ is the

input photon operator satisfying the commutation relation $[a_{in}(t), a_{in}^{\dagger}(t')] = \delta(t - t')$, and $|vac\rangle$ is the vacuum state of the optical field [18]. If the atom is prepared in $|0\rangle$ (or in $|1\rangle$) initially, then the photon will trigger the transition $|0\rangle \leftrightarrow |e\rangle$ (or $|1\rangle \leftrightarrow |e\rangle$). The quantum Langevin equations are

$$\begin{aligned}\dot{a}(t) &= -\,[i(\omega_c - \omega_{\mathrm{p}}) + \frac{\kappa}{2}]a(t) - g\sigma_-(t) - \sqrt{\kappa}a_{\mathrm{in}}(t), \\ \dot{\sigma}_-(t) &= -\,[i(\omega_0 - \omega_{\mathrm{p}}) + \frac{\gamma}{2}]\sigma_-(t) - g\sigma_z(t)a(t) + \sqrt{\gamma}\sigma_z(t)b_{\mathrm{in}}(t),\end{aligned} \tag{1}$$

where κ and γ are the cavity damping rate and atomic spontaneous emission rate, respectively. $b_{in}(t)$ is vacuum input field felt by the atom. The contribution of vacuum field $b_{in}(t)$ is negligible because it is much less than the one of the photon pulse $a_{in}(t)$. The output field $a_{out}(t)$ relates to the input field by the intracavity field as $a_{out}(t) = a_{in}(t) + \sqrt{\kappa}a(t)$ [19]. Assume that κ is large enough to ensure the atom initially prepared in the ground state is only virtually excited by the photon, so we can take $\langle\sigma_z(t)\rangle = -1$. Also under this large κ limit, we can adiabatically eliminate the intracavity mode from the set of quantum Langevin equations and arrive consequently at the input-output relation as

$$r(\omega_p) = \frac{[i(\omega_c - \omega_p) - \frac{\kappa}{2}][i(\omega_0 - \omega_p) + \frac{\gamma}{2}] + g^2}{[i(\omega_c - \omega_p) + \frac{\kappa}{2}][i(\omega_0 - \omega_p) + \frac{\gamma}{2}] + g^2}. \tag{2}$$

where $r(\omega_p) = \frac{a_{out}(t)}{a_{in}(t)}$ is the reflection coefficient of the photon to the atom-cavity system. On the other hand, if the atom is prepared in $|1\rangle$ (or $|0\rangle$) initially under the condition that a L- (or R-) circularly polarized photon pulse is input in the cavity, then no transition would be triggered. In other words, the photon only feels an empty cavity, for which the input-output relation corresponds to Eq. (2) with $g = 0$ [19], i.e.

$$r_0(\omega_p) = \frac{i(\omega_c - \omega_p) - \frac{\kappa}{2}}{i(\omega_c - \omega_p) + \frac{\kappa}{2}}. \tag{3}$$

The complex reflection coefficients (2) and (3) show that the reflected photon experiences an absorption as well as a phase shift denoted by $e^{i\phi}$ and $e^{i\phi_0}$, respectively. However, under the practical experimental condition, i.e., the strong κ and weak g and γ [13], it has been verified that the absolute values of $r(\omega_p)$ and $r_0(\omega_p)$ are only slightly deviated from unity [20, 15]. This implies that the photon experiences a very weak absorption, and thereby we may approximately consider that the output photon only experiences a pure phase shift without any absorption.

With this basic input-output relation, the Faraday rotation can be derived readily when a linearly polarized photon is input into the cavity. Consider the input pulse is linearly polarized in $|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ and the atom initially in $|0\rangle$, the output photon would have a rotation in the polarization. In this case, the $|L\rangle$ component of the photon virtually triggers the transition of the atom from $|0\rangle$ to $|e\rangle$, and thus experiences a phase shift $e^{i\phi}$ obeying Eq. (2). In contrast, the $|R\rangle$ component of the photon only feels an empty cavity, which yields a phase shift $e^{i\phi_0}$ obeying Eq. (3). So the output pulse is

$$|\Psi_{out}\rangle_- = \frac{1}{\sqrt{2}}(e^{i\phi}|L\rangle + e^{i\phi_0}|R\rangle). \tag{4}$$

This also implies that the polarization direction of the reflected photon rotates an angle $\Theta_F^- = \frac{\phi_0 - \phi}{2}$ with respect to the input one, called Faraday rotation [16]. Similarly, if the atom is initially prepared in $|1\rangle$, then only the R circularly polarized photon could sense the atom, while the L circularly polarized photon only feels the empty cavity. So we have

$$|\Psi_{out}\rangle_+ = \frac{1}{\sqrt{2}}(e^{i\phi_0}|L\rangle + e^{i\phi}|R\rangle), \quad (5)$$

corresponding to a Faraday rotation with an angle $\Theta_F^+ = \frac{\phi - \phi_0}{2}$.

3. The realization of multipartite-state teleportation via Faraday rotation

In what follows, we show the teleportation of a multipartite state by above Faraday rotations. As plotted in Fig. 1 (c), to construct the entanglement between Alice and Bob, Bob has to first input N single-photon pulses into his cavities to produce N pairs of entangled states between the photons and atoms using the Faraday rotations of the photons. Then the emitted photons fly to Alice through the fiber. Once Alice collects the photons in her cavities, the entanglement between Alice and Bob has been established. We emphasize that the photon in our scheme acts dual roles: On the one hand, it acts as a bus to distribute entanglement; On the other hand, it also acts as a component of the entangled pair to implement the teleportation.

3.1. The case of bipartite state

To demonstrate our scheme specifically, we will take $N = 2$ below as an example. Suppose the quantum state to be teleported in Alice's hands is [21]

$$|\varphi\rangle = \alpha\,|01\rangle + \beta\,|10\rangle + \zeta\,|00\rangle + \delta\,|11\rangle\,, \quad (6)$$

where α, β, ζ, and δ are unknown parameters with $\alpha^2 + \beta^2 + \zeta^2 + \delta^2 = 1$. In the following we give explicitly the steps for accomplishing the quantum teleportation.

The first step: Establishment of the quantum channels. Bob inputs two linearly polarized single-photon pulses in $|\Psi\rangle_i = \frac{1}{\sqrt{2}}(|L\rangle_i + |R\rangle_i)$ generated from the single-photon source into two cavities at his side. The states of the atoms confined in Bob's cavities are initially $|\psi\rangle_i = \frac{1}{\sqrt{2}}(|0\rangle_i + |1\rangle_i)$, $(i = 1, 2)$. According to the input-output relation, the atoms and the photons are entangled due to the Faraday rotation,

$$|\Psi\rangle_i\,|\psi\rangle_i \rightarrow \frac{1}{\sqrt{2}}[|0\rangle_i|\Psi_{out}\rangle_{-i} + |1\rangle_i|\Psi_{out}\rangle_{+i}]. \quad (7)$$

Then after Alice collects the two output photons via two fibers, two quantum channels are thus established. One can see that $|\Psi_{out}\rangle_+$ and $|\Psi_{out}\rangle_-$ are orthogonal when $\phi - \phi_0 = \pi/2$. It means that under this condition, the quantum channel characterized by the right side of Eq. (7) is maximally entangled. In practice, this condition can be easily satisfied experimentally. Using the parameters in Ref. [13], $\omega_0 = \omega_c$, $\omega_p = \omega_c - \frac{\kappa}{2}$, and $g = \frac{\kappa}{2}$, we can verify from Eqs. (2) and (3) that $\phi = \pi$ and $\phi_0 = \pi/2$. The Faraday rotation under the same condition has been used to realize the universal quantum gate,

the entanglement generation between remote atoms and its conversion to the flying photons in Ref. [15, 22].

The second step: Realization of the Bell-state measurement. In the standard protocol of quantum teleportaion, a necessary step is the Bell-state measurement on the atom and one of the entangled pair of the quantum channel in Alice's side [1]. The measurement would collapse the state of the total system in one of its four superposition components. Experimentally, this Bell-state measurement is ordinarily realized by the CNOT operation plus the product-state measurement. In the following we show the CNOT operation can actually be replaced by the Faraday rotation in Alice's side, which much relaxes the experimental difficulty to do the CNOT operation. After collecting the photons, Alice feeds the photons to her cavities. The Faraday rotation makes the two atoms at Alice's hands entangled with the two quantum channels. So the state of the entire system could be written as,

$$
\begin{aligned}
&|\varphi\rangle \prod_{i=1,2} \frac{1}{\sqrt{2}}[|0\rangle_i|\Psi_{out}\rangle_{-i} + |1\rangle_i|\Psi_{out}\rangle_{+i}] \rightarrow \\
&\frac{1}{4}[|LL\rangle\left(-i\alpha\left|01\right\rangle - i\beta\left|10\right\rangle + \zeta\left|00\right\rangle - \delta\left|11\right\rangle\right) \\
&\times\left(|00\rangle_{12} - i\,|01\rangle_{12} - i\,|10\rangle_{12} - |11\rangle_{12}\right) \\
&+|LR\rangle\left(\alpha\left|01\right\rangle - \beta\left|10\right\rangle - i\zeta\left|00\right\rangle - i\delta\left|11\right\rangle\right) \\
&\times\left(-i\,|00\rangle_{12} + |01\rangle_{12} - |10\rangle_{12} - i\,|11\rangle_{12}\right) \\
&+|RL\rangle\left(-\alpha\left|01\right\rangle + \beta\left|10\right\rangle - i\zeta\left|00\right\rangle - i\delta\left|11\right\rangle\right) \\
&\times\left(-i\,|00\rangle_{12} - |01\rangle_{12} + |10\rangle_{12} - i\,|11\rangle_{12}\right) \\
&+|RR\rangle\left(-i\alpha\left|01\right\rangle - i\beta\left|10\right\rangle - \zeta\left|00\right\rangle + \delta\left|11\right\rangle\right) \\
&\times\left(-\,|00\rangle_{12} - i\,|01\rangle_{12} - i\,|10\rangle_{12} + |11\rangle_{12}\right)],
\end{aligned}
\tag{8}
$$

where the kets with and without subscripts correspond to the atomic states at Bob's and Alice's hands, respectively. We have used $\phi = \pi$ and $\phi_0 = \pi/2$ with the practical parameters in Ref. [13].

Next, Alice makes Hadamard gates on the photons and atoms at her side. The photonic Hadamard gating is realized by a quarter-wave plate (QWP), which makes $|R\rangle \rightarrow \frac{|L\rangle - |R\rangle}{\sqrt{2}}$ and $|L\rangle \rightarrow \frac{|L\rangle + |R\rangle}{\sqrt{2}}$. Then the right-hand side of Eq. (8) is converted into $\sum_{i,j;m,n} |ij, mn\rangle |f_{ij,mn}\rangle_{12}$, where $|ij\rangle$, $|mn\rangle$, and $|f_{ij,mn}\rangle_{12}$ denote the photon, and Alice's and Bob's atomic states, respectively, as shown in Table 1.

Finally, Alice performs measurement on the states of the photons and the atoms at her side, followed by the collapse of the atomic state at Bob's side to one of the corresponding components in above superposition.

The third step: Bob's recovery operations. Based on the message (i, j, m, n) from Alice via the classical channel about her measurement, Bob could deterministically recover the unknown state only by some local operations $M_{ij,mn}$ on his atoms, as listed in Table 1.

Table 1. The superposition components of the final state and Bob's corresponding operations when $N = 2$.

$\|ij, mn\rangle$	$\|f_{ij,mn}\rangle_{12}$	$M_{ij,mn}$
$\|LL\rangle\|00\rangle$	$-\alpha\|10\rangle - \beta\|01\rangle - \zeta\|11\rangle - \delta\|00\rangle$	$-\sigma_x^{(1)} \otimes \sigma_x^{(2)}$
$\|LL\rangle\|11\rangle$	$\alpha\|10\rangle + \beta\|01\rangle - \zeta\|11\rangle - \delta\|00\rangle$	$\sigma_y^{(1)} \otimes \sigma_y^{(2)}$
$\|LL\rangle\|01\rangle$	$-\alpha\|10\rangle + \beta\|01\rangle + \zeta\|11\rangle - \delta\|00\rangle$	$\sigma_x^{(1)} \otimes \sigma_y^{(2)}$
$\|LL\rangle\|10\rangle$	$\alpha\|10\rangle - \beta\|01\rangle + \zeta\|11\rangle - \delta\|00\rangle$	$\sigma_y^{(1)} \otimes \sigma_x^{(2)}$
$\|LR\rangle\|00\rangle$	$i\alpha\|11\rangle - i\beta\|00\rangle - i\zeta\|10\rangle + i\delta\|01\rangle$	$\sigma_x^{(1)} \otimes \sigma_z^{(2)}$
$\|LR\rangle\|11\rangle$	$-i\alpha\|11\rangle + i\beta\|00\rangle - i\zeta\|10\rangle + i\delta\|01\rangle$	$\sigma_y^{(1)} \otimes I^{(2)}$
$\|LR\rangle\|01\rangle$	$i\alpha\|11\rangle + i\beta\|00\rangle + i\zeta\|10\rangle + i\delta\|01\rangle$	$\sigma_x^{(1)} \otimes I^{(2)}$
$\|LR\rangle\|10\rangle$	$-i\alpha\|11\rangle - i\beta\|00\rangle + i\zeta\|10\rangle + i\delta\|01\rangle$	$\sigma_y^{(1)} \otimes \sigma_z^{(2)}$
$\|RL\rangle\|00\rangle$	$i\alpha\|00\rangle + i\beta\|11\rangle - i\zeta\|01\rangle + i\delta\|10\rangle$	$\sigma_z^{(1)} \otimes \sigma_x^{(2)}$
$\|RL\rangle\|11\rangle$	$i\alpha\|00\rangle - i\beta\|11\rangle - i\zeta\|01\rangle + i\delta\|10\rangle$	$I^{(1)} \otimes \sigma_y^{(2)}$
$\|RL\rangle\|01\rangle$	$-i\alpha\|00\rangle - i\beta\|11\rangle + i\zeta\|01\rangle + i\delta\|10\rangle$	$\sigma_z^{(1)} \otimes \sigma_y^{(2)}$
$\|RL\rangle\|10\rangle$	$i\alpha\|00\rangle + i\beta\|11\rangle + i\zeta\|01\rangle + i\delta\|10\rangle$	$I^{(1)} \otimes \sigma_x^{(2)}$
$\|RR\rangle\|00\rangle$	$-\alpha\|00\rangle - \beta\|11\rangle + \zeta\|01\rangle + \delta\|10\rangle$	$\sigma_z^{(1)} \otimes \sigma_z^{(2)}$
$\|RR\rangle\|11\rangle$	$\alpha\|00\rangle + \beta\|11\rangle + \zeta\|01\rangle + \delta\|10\rangle$	$I^{(1)} \otimes I^{(2)}$
$\|RR\rangle\|01\rangle$	$-\alpha\|00\rangle + \beta\|11\rangle - \zeta\|01\rangle + \delta\|10\rangle$	$\sigma_z^{(1)} \otimes I^{(2)}$
$\|RR\rangle\|10\rangle$	$\alpha\|00\rangle - \beta\|11\rangle - \zeta\|01\rangle + \delta\|10\rangle$	$I^{(1)} \otimes \sigma_z^{(2)}$

3.2. The case of tripartite state

With more single-photon pulses, we may generalize our scheme straightforwardly to teleportation of bigger entangled states. For simplicity, we take a tripartite state as an example. Suppose the teleported tripartite state to be of the GHZ-like form,

$$|\varphi\rangle = \alpha|000\rangle + \delta|111\rangle. \tag{9}$$

In the first step, Bob inputs three photons in his cavities. The input-output process in his side makes the photons entangled with the atoms in each of the cavities. After Alice collects the three photons, three quantum channels have thus been established. In the second step, the Faraday rotation in Alice's side makes the state of the whole system transform into

$$\begin{aligned}
&|\varphi\rangle \prod_{i=1,2,3} \frac{1}{\sqrt{2}}[|0\rangle_i|\Psi_{out}\rangle_{-i} + |1\rangle_i|\Psi_{out}\rangle_{+i}] \rightarrow \\
&\frac{1}{8}[(-|0\rangle_1 + i|1\rangle_1)(-|0\rangle_2 + i|1\rangle_2)(-|0\rangle_3 + i|1\rangle_3)|LLL\rangle_{123}(-\alpha|000\rangle - i\delta|111\rangle) \\
&+ (-|0\rangle_1 + i|1\rangle_1)(-|0\rangle_2 + i|1\rangle_2)(i|0\rangle_3 - |1\rangle_3)|LLR\rangle_{123}(i\alpha|000\rangle + \delta|111\rangle) \\
&+ (-|0\rangle_1 + i|1\rangle_1)(i|0\rangle_2 - |1\rangle_2)(-|0\rangle_3 + i|1\rangle_3)|LRL\rangle_{123}(i\alpha|000\rangle + \delta|111\rangle) \\
&+ (-|0\rangle_1 + i|1\rangle_1)(i|0\rangle_2 - |1\rangle_2)(i|0\rangle_3 - |1\rangle_3)|LRR\rangle_{123}(\alpha|000\rangle + i\delta|111\rangle) \\
&+ (i|0\rangle_1 - |1\rangle_1)(-|0\rangle_2 + i|1\rangle_2)(-|0\rangle_3 + i|1\rangle_3)|RLL\rangle_{123}(i\alpha|000\rangle + \delta|111\rangle) \\
&+ (i|0\rangle_1 - |1\rangle_1)(-|0\rangle_2 + i|1\rangle_2)(i|0\rangle_3 - |1\rangle_3)|RLR\rangle_{123}(\alpha|000\rangle + i\delta|111\rangle)
\end{aligned}$$

Table 2. The superposition components of the final state and Bob's corresponding operations when $N = 3$.

$\vert ijk\rangle$	$\vert lmn\rangle$	$\vert f_{ijk,lmn}\rangle_{123}$	$M_{ijk,lmn}$
$\vert LLL\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$i\alpha\vert 111\rangle + i\delta\vert 000\rangle$	$\sigma_x \otimes \sigma_x \otimes \sigma_x$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$i\alpha\vert 111\rangle - i\delta\vert 000\rangle$	$\sigma_x \otimes \sigma_x \otimes \sigma_y$
$\vert RRR\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$\alpha\vert 000\rangle - \delta\vert 111\rangle$	$I \otimes I \otimes \sigma_z$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$\alpha\vert 000\rangle + \delta\vert 111\rangle$	$I \otimes I \otimes I$
$\vert LLR\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-\alpha\vert 110\rangle + \delta\vert 001\rangle$	$\sigma_x \otimes \sigma_x \otimes \sigma_z$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-\alpha\vert 110\rangle - \delta\vert 001\rangle$	$\sigma_x \otimes \sigma_x \otimes I$
$\vert LRL\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-\alpha\vert 101\rangle + \delta\vert 010\rangle$	$\sigma_x \otimes \sigma_z \otimes \sigma_x$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-\alpha\vert 101\rangle - \delta\vert 010\rangle$	$\sigma_x \otimes I \otimes \sigma_y$
$\vert RLL\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-\alpha\vert 011\rangle + \delta\vert 100\rangle$	$\sigma_z \otimes \sigma_x \otimes \sigma_x$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-\alpha\vert 011\rangle - \delta\vert 100\rangle$	$I \otimes \sigma_x \otimes \sigma_y$
$\vert LRR\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-i\alpha\vert 100\rangle - i\delta\vert 011\rangle$	$\sigma_x \otimes I \otimes I$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-i\alpha\vert 100\rangle + i\delta\vert 011\rangle$	$\sigma_y \otimes I \otimes I$
$\vert RLR\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-i\alpha\vert 010\rangle - i\delta\vert 101\rangle$	$I \otimes \sigma_x \otimes I$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-i\alpha\vert 010\rangle + i\delta\vert 101\rangle$	$I \otimes \sigma_y \otimes I$
$\vert RRL\rangle$	$\vert 000\rangle, \vert 011\rangle, \vert 101\rangle, \vert 110\rangle$	$-i\alpha\vert 001\rangle - i\delta\vert 110\rangle$	$I \otimes I \otimes \sigma_x$
	$\vert 001\rangle, \vert 010\rangle, \vert 100\rangle, \vert 111\rangle$	$-i\alpha\vert 001\rangle + i\delta\vert 110\rangle$	$I \otimes I \otimes \sigma_y$

$$+ (i|0\rangle_1 - |1\rangle_1)(i|0\rangle_2 - |1\rangle_2)(-|0\rangle_3 + i|1\rangle_3)|RRL\rangle_{123}(\alpha|000\rangle + i\delta|111\rangle)), \quad (10)$$

which could be further transformed into the final form $\sum_{ijk,lmn} |ijk, lmn\rangle|f_{ijk,lmn}\rangle_{123}$ by local Hadamard operations applied to the photonic and to the atomic states in Alice's side. The explicit form of the superposition components in the final state is listed in Table 2. Finally, Alice performs measurement on the state of photons and the atoms and then tells Bob her results. In the last step, Bob has to choose local operations $M_{ijk,lmn}$ appropriately, as shown in Table 2, to recover the unknown teleported state on his atoms.

4. Discussions and summary

Our scheme, involving only virtual excitation of the atoms, is robust to spontaneous emission. Besides, as cavity decay has been considered in the reflection coefficient Eq. (2), our scheme could work with bad cavities in the case of large cavity decay and/or weak couplings. The dominant source of error in our scheme is the photon loss due to the cavity mirror absorption and scattering, the fiber absorption, and the inefficiency of the detector. Nevertheless, since the accomplishment of our scheme relies on the successful detection of the photons, the photon loss does not affect the fidelity, but only the efficiency.

The Hadamard gate operations on the atoms could be done using Raman

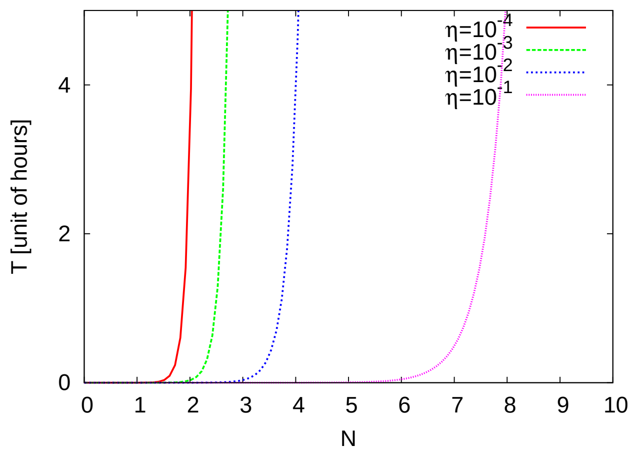

Figure 2. (Color online). The implementation time of the teleportation as a function of the number of the parties involved, with different single-photon detector inefficiency η.

configuration by two polarized lasers detuned from the $|0\rangle \leftrightarrow |e\rangle$ and $|1\rangle \leftrightarrow |e\rangle$ transitions. In addition, the measurement of the atomic states could be carried out by a resonant laser. For example, the successful detection of a leaking photon from the cavity after the radiation on the atom by a L-polarized laser means the population in the atomic state $|0\rangle$. Otherwise, we may try to detect the atomic state $|1\rangle$ using a R-polarized laser.

We argue that the implementation of our scheme heavily depends on the imperfection rate due to relevant techniques. Under currently available technique, our scheme could be accomplished within a finite time if the qubit number N is not very large. Specifically, supposing that the failure rate associated with the decay of the virtual atomic excitation is about 2%, the current dark count rate of the single-photon detector yields the inefficiency of 10^{-4}, and other imperfection rate due to photon loss is about 6%, we thus have the success rate to be $[(1-2\%)^2 \times 10^{-4} \times (1-6\%)]^N$. Fortunately, thanks to the highly efficient single-photon generator producing 10000 photons every second [23], we may accomplish the teleportation in finite time. For example, a successful teleportation of a two-qubit state takes about three hours.

This time-consuming implementation is mainly resulted from the low efficiency of currently available single-photon detectors, which is also a problem in any of the previously published schemes using photon interference. To shorten the implementation time, on the one hand, we should increase the resource. For example, if Bob possesses N sets of single-photon source, then the implementation time would be reduced by N times. On the other hand, if the efficiency of the single-photon detector could be enhanced, then our implementation time would be much reduced by several degrees of magnitude. A simple estimate of the implementation time of our scheme is plotted in Fig. 2 with respect to different imperfect factors.

In comparison to previous proposals for teleportation considering virtually excited cavity modes [10] and cavity decay [12], our scheme is advantageous to work very well not only in the case of the bad cavities, but also in perfect and deterministic fashion. Using photons as flying qubits to transfer quantum information, it is more suitable for long-distance communication compared to the schemes in Ref. [10, 11]. Moreover, our scheme using bipartite entanglement as quantum channels is more robust to decoherence than others based on multipartite entanglement [9].

In conclusions, we have proposed a practical scheme for teleportation of an arbitrary N-partite pure state using the Faraday rotation. Besides the use in atomic system, our idea could also be applied to quantum-dot system after minor modification: Replacing the atomic excitations by the excitonic ones [14]. As it needs no CNOT operation, only involves product-state measurements, and works perfectly and deterministically in low-Q cavities, our scheme would be useful for building quantum network and for scalable quantum computation using currently achieved microtoroidal resonators [13] or single-sided cavities [17].

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References

[1] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895.

[2] Pan J-W, Mattle K, Eibl M, Weinfurter H, Zeilinger A and Bouwmeester D 1997 *Nature* **390** 575; Boschi D, Branca S, DeMartini F, Hardy L and Popescu S 1998 *Phys. Rev. Lett.* **80** 1121; Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Körber T W, Becher C, Schmidt-Kaler F, James D F V and R. Blatt 2004 *Nature* **429** 734; Barrett M D, Chiaverini J, Schaetz T, Britton J, Itano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737.

[3] Sherson J F, Krauter H, Olsson R K, Julsgaard B, Hammerer K, Cirac I and Polzik E S 2006 *Nature* **443** 557.

[4] Briegel H-J, Dür W, Cirac J I and Zoller P 1998 *Phys. Rev. Lett.* **81** 5932.

[5] Kimble H J 2008 *Nature* **453** 1023.

[6] Gottesman D and Chuang I L 1999 *Nature* **402** 390.

[7] Lee J, Min H and Oh S D 2002 *Phys. Rev.* A **66** 052318.

[8] Fang J, Lin Y, Zhu S and Chen X 2003 *Phys. Rev.* A **67** 014305.

[9] Yeo Y and Chua W K 2006 *Phys. Rev. Lett.* **96** 060502; Rigolin G 2005 *Phys. Rev.* A **71** 032303; Muralidharan S and Panigrahi P K 2008 *Phys. Rev.* A **77** 032321.

[10] Ye L and Guo G-C 2004 *Phys. Rev.* A **70** 054303.

[11] Zheng S-B 2004 *Phys. Rev.* A **69** 064302; Cardoso W B, Avelar A T, Baseia B and de Almeida N G 2005 *Phys. Rev.* A **72** 045802.

[12] Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158; Zheng S-B and Guo G-C 2006 *Phys. Rev.* A **73** 032329.

[13] Dayan B, Parkins A S, Aoki T, Ostby E P, Vahala K J and Kimble H J 2008 *Science* **319** 1062.

[14] Leuenberger M N, Flatté M E and Awschalom D D 2005 *Phys. Rev. Lett.* **94** 107401.

[15] An J-H, Feng M and Oh C H 2009 *Phys. Rev.* A **79** 032303.
[16] Julsgaard B, Kozhekin A and Polzik E S 2001 *Nature* **413** 400.
[17] Turchette Q A, Hood C J, Lange W, Mabuchi H and Kimble H J 1995 *Phys. Rev. Lett.* **75** 4710.
[18] Duan L-M and Kimble H J 2004 *Phys. Rev. Lett.* **92** 127902.
[19] Walls D F and Milburn G J 1994 *Quantum Optics* (Springer-Verlag, Berlin).
[20] Hu C Y, Young A, O'Brien J L, Munro W J and Rarity J G 2008 *Phys. Rev.* B **78** 085307.
[21] The initially entangled state could be prepared via Faraday rotation by connecting two of the cavities in Alice's side [15]: The single photon input-output process regarding Alice's cavities leads the state to that in Eq. (12) in Ref. [15]. Then a detection of the h-polarized photon would achieve the desired state. Meanwhile, we have to mention that, as we focus on how to teleport the entangled state, there is no consideration in Fig. 1 (c) for the generation of the entangled state to be teleported.
[22] Chen Q and Feng M 2009 *Phys. Rev.* A **79** 064304.
[23] Hijlkema M, Weber B, Specht H P, Webster S C, Kuhn A and Rempe G 2007 *Nat. Phys.* **3** 253.